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Spinorial cohomology of abelian $d = 10$ super-Yang-Mills at $\mathcal{O}(\alpha'^3)$

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Abstract: We compute the spinorial cohomology of ten-dimensional abelian SYM at order α'^3 and we find that it is trivial. Consequently, linear supersymmetry *alone* excludes the presence of α'^3-order corrections. Our result lends support to the conjecture that there may be a unique supersymmetric deformation of ordinary ten-dimensional abelian SYM.

Keywords: D-branes, Extended Supersymmetry, Supersymmetric Effective Theories, Superspaces.

Contents

1. Introduction

It was realized some time ago that the Born-Infeld action [1] (see [2] for a review) occurs in string theory as the tree-level open-string effective action in the limit of slowly varying field-strengths [3, 4]. After the discovery of D-branes [5], the BI action in $p+1$ spacetime dimensions was re-interpreted as the low-energy effective theory on the world-volume of a single p-brane [6]. The supersymmetrization of the BI in ten dimensions was obtained in [7] by gauge-fixing the κ-symmetric D-brane actions of [8]–[11]. It has been subsequently re-derived using the superembedding formalism [12].[1]

There have been claims in the literature that due to the restrictive form of supersymmetry in ten dimensions, the abelian supersymmetric BI may be the unique supersymmetric deformation of ordinary abelian SYM, in the limit of slowly-varying field-strengths. There is evidence in favor of this claim coming from the Seiberg-Witten map [14] and the "form-invariance" of the BI action under it [15, 16], as well as from studying the deformations of certain BPS configurations of D-branes [17]. A stronger form of this conjecture would be that there is a unique supersymmetric deformation of ordinary abelian SYM, even when the slowly-varying field-strength assumption $\partial F = 0$ is lifted. The abelian BI would then be the $\partial F \to 0$ limit of that unique deformation.

In this paper we lend support to this conjecture: By using superspace techniques, we investigate the space of all possible supersymmetric deformations of ten-dimensional abelian SYM at order α'^3 and we find that it is zero-dimensional. Consequently, to this order in α', the ten-dimensional abelian BI is indeed the unique supersymmetric deformation of ordinary abelian SYM.

A related result was proven some time ago [18] (see also [19]) in the context of open-string theory. One of the implications of this reference is that at tree level in the string coupling constant there are no derivative corrections to the ten-dimensional abelian supersymmetric BI at order α'^3, in contrast to the bosonic case. Our result makes no reference to string theory. The only input is ten-dimensional *linear* supersymmetry.

[1] See [13] for a recent discussion in the context of the pure-spinor formalism of open superstrings.

The superspace Bianchi identities for ten-dimensional SYM were solved in [20], using only the standard conventional constraint [21, 22]. It was observed that the deformations of ordinary SYM are controlled (parameterized) by a chiral five-form J_5 (see [23, 24] for earlier work). This object may be thought of as a composite operator made of the SYM fields λ^α (the gaugino) and F_{ab} (the gauge-invariant field-strength). One can obtain an α' expansion, by considering the most general ansatz for J_5 at each order in α'. By inserting the ansatz into the Bianchi identities, the tower of α' corrections is generated. This procedure was carried out to order α'^2 in [25].

However, as noted in [20], not every J_5 is consistent with the Bianchi identities. The consistency condition $D_2 J_5 = 0$ has to be satisfied, where the action of D_2 is given by a spinor superderivative and a subsequent projection onto the highest representation. Moreover, some part of J_5 will generically be removable by redefinitions of the spinor superfield A_α. These redefinitions shift J_5 by an amount $D_1 A_\alpha$, where the action of D_1 is again given by a spinor superderivative and a projection onto the highest representation. The effective deformations of the theory are therefore given by the most general J_5 which cannot be written as $D_1 A_\alpha$ and obeys $D_2 J_5 = 0$. As remarked in [26, 27] the operators $D_{1,2}$ satisfy a nilpotency condition

$$D_2 \circ D_1 = 0 \tag{1.1}$$

and define a spinorial cohomology SH,

$$SH = \frac{\mathrm{Ker}(D_2)}{\mathrm{Im}(D_1)}\,. \tag{1.2}$$

The supersymmetric deformations of the theory are in one-to-one correspondence with elements of SH.

We should emphasize that the *only* assumption in this procedure is that the theory possesses linear supersymmetry. This is to be contrasted with the superembedding formalism or κ-symmetry, which "know" about nonlinear supersymmetry as well and are therefore more restrictive in principle.

In this paper we compute SH for $N = 1$, $d = 10$ abelian SYM at order α'^3 , counting orders of α' relative to the F^2 term. We find that $SH = 0$ and in view of the remark following (1.2), we conclude that there is no possible supersymmetric deformation of ordinary abelian SYM at this order in α'.

The next section is a brief review of SYM in the language of ten-dimensional superspace. Section 3 contains the computation of spinorial cohomology at order α'^3. We conclude with some discussion in section 4.

2. Review of $d = 10$ SYM

This section contains a brief review of ten-dimensional SYM in superspace language, mainly for establishing notation and conventions. For a more extensive discussion the reader is referred to [20, 25].

Ten-dimensional SYM contains a gauge superfield (A_α, A_a). Ordinary SYM is obtained by imposing the constraint [28],

$$F_{\alpha\beta} = 0\,. \tag{2.1}$$

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The vector part of this constraint, $\Gamma_a^{\alpha\beta}F_{\alpha\beta}=0$, is the so called conventional constraint [21, 22] and serves the purpose of eliminating a redundant vector potential sitting at first level in the θ expansion of A_α. In order to relax (2.1) one expresses $F_{\alpha\beta}$ in terms of an anti-self-dual five-form J_5,

$$F_{\alpha\beta}=\frac{1}{5!}\Gamma_{\alpha\beta}^{a_1\ldots a_5}J_{a_1\ldots a_5}\,. \tag{2.2}$$

Note that the conventional constraint is still implemented.

When (2.2) is plugged into the superspace Bianchi identities, the (relaxed) equations of motion for the gaugino λ^α and the gauge-invariant field strength F_{ab} are obtained in terms of fields appearing at various levels in the θ expansion of J_5. The superspace Bianchi identities with the relaxed constraint (2.2) were solved in [20]. In the limit $J_5\to 0$ one recovers ordinary SYM. By expanding J_5 in α', one generates the tower of α' corrections to ordinary SYM. As mentioned in the introduction, the Bianchi identities imply that J_5 satisfies,

$$D_2J_{a_1\ldots a_5}:=D_\alpha J_{a_1\ldots a_5}|=0\,, \tag{2.3}$$

where by | we denote the projection onto the anti-self-dual (00030) part. An explicit expression for the projection is given below in equation (3.9). Moreover, a redefinition δA_α of the spinor superpotential shifts J_5 by an amount proportional to $D_1\delta A_\alpha$, where,

$$D_1\delta A_\alpha:=\frac{1}{5!}\Gamma^{\alpha\beta}_{a_1\ldots a_5}D_\alpha\delta A_\beta\,. \tag{2.4}$$

Supersymmetric deformations of the theory are elements of the spinorial cohomology (1.2).

Let us conclude this section with some comments on dimensions and representations. The mass dimensions of the various fields are as follows:

$$[A_\alpha]=\frac{1}{2}\,;\qquad [\lambda^\alpha]=\frac{3}{2}\,;\qquad [F_{ab}]=2\,;\qquad [J_{a_1\ldots a_5}]=1\,, \tag{2.5}$$

while $[\alpha']=-2$. In terms of representations of $so(1,9)\approx D_5$ the fields transform as

$$A_\alpha\sim(00010)\,;\qquad \lambda^\alpha\sim(00001)\,;\qquad F_{ab}\sim(01000)\,;\qquad J_{a_1\ldots a_5}\sim(00020)\,. \tag{2.6}$$

Our convention is that (00020) is anti-self-dual whereas (00002) is self-dual. Using the lowest-order equations of motion

$$\Gamma^aD_a\lambda=0\,;\qquad D^aF_{ab}=0\,, \tag{2.7}$$

it is not difficult to see that

$$D_{a_1}\ldots D_{a_n}\lambda\sim(n0001)\,;\qquad D_{a_1}\ldots D_{a_n}F_{ab}\sim(n1000)\,. \tag{2.8}$$

Finally, the action of the spinor superderivative to lowest order in α' is given by,

$$D_\alpha\lambda^\beta=\frac{1}{2}(\Gamma^{ab})_\alpha{}^\beta F_{ab}\,;\qquad D_\alpha F_{ab}=2(\Gamma_{[a}D_{b]}\lambda)_\alpha\,. \tag{2.9}$$

3. Spinorial cohomology at $\mathcal{O}(\alpha'^3)$

In order to compute the spinorial cohomology at $\mathcal{O}(\alpha'^3)$, we have to carry out the following procedure: a) Write down the most general anti-self-dual five-form J_5 and spinor-potential redefinition δA_α which are allowed by dimensional analysis and are of the form α'^3 times a product of λ^α's and F_{ab}'s. b) Compute the action of $D_{1,2}$. c) Compute the spinorial cohomology using (1.2). Let us note that in carrying out steps a) – c) above, one need only use the *lowest-order* equations (2.7), (2.9). This is because δA_α, J_5 already contain one power of α'^3.

Taking the last paragraph of the previous section into account, we find that the most general J_5 at order α'^3 is of the form

$$J_{a_1\dots a_5} = \sum_{i=1}^{10} b_i B^{(i)}_{a_1\dots a_5}\,, \tag{3.1}$$

where $b_1, \dots b_{10} \in \mathbb{R}$. Schematically, $B^{(1)} \dots B^{(10)}$ are given by the projections of the following products of irreducible representations onto (00020):

$$\begin{aligned}
B^{(1)} &\sim F^2\lambda^2 \sim (01000)^2_s \otimes (00001)^2_a \\
B^{(2)} &\sim \lambda^3 D\lambda \sim (00001)^3_a \otimes (10001) \\
B^{(3)} &\sim F^2 DF \sim (01000)^2_s \otimes (11000) \\
B^{(4,5,6)} &\sim F(D\lambda)^2 \sim (01000) \otimes (10001)^2 \\
B^{(7)} &\sim F\lambda D^2\lambda \sim (01000) \otimes (00001) \otimes (20001) \\
B^{(8,9,10)} &\sim DF\lambda D\lambda \sim (11000) \otimes (00001) \otimes (10001)\,.
\end{aligned} \tag{3.2}$$

More explicitly (antisymmetrization in $a_1 \dots a_5$ is implied on the right-hand side),

$$\begin{aligned}
B^{(1)}_{a_1\dots a_5} &= (\lambda\Gamma_{a_2\dots a_5}{}^{ijk}\lambda)F_{a_1 i}F_{jk} - \text{dual} \\
B^{(2)}_{a_1\dots a_5} &= 2(\lambda\Gamma_{a_1}D_{a_2}\lambda)(\lambda\Gamma_{a_3a_4a_5}\lambda) - \text{dual} \\
B^{(3)}_{a_1\dots a_5} &= F_{a_1a_2}F_{a_3 i}D^i F_{a_4a_5} - \text{dual} \\
B^{(4)}_{a_1\dots a_5} &= (D_{a_1}\lambda\Gamma_{a_2a_3a_4}D^i\lambda)F_{a_5 i} - \text{dual} \\
B^{(5)}_{a_1\dots a_5} &= (D^i\lambda\Gamma_{a_1a_2a_3}D_i\lambda)F_{a_4a_5} - \text{dual} \\
B^{(6)}_{a_1\dots a_5} &= (D_{a_1}\lambda\Gamma_{a_2}D_{a_3}\lambda)F_{a_4a_5} - \text{dual} \\
B^{(7)}_{a_1\dots a_5} &= (D^i D_{a_1}\lambda\Gamma_{a_2a_3a_4}\lambda)F_{a_5 i} - \text{dual} \\
B^{(8)}_{a_1\dots a_5} &= (\lambda\Gamma_{a_1a_2 i}D_{a_3}\lambda)D^i F_{a_4a_5} - \text{dual} \\
B^{(9)}_{a_1\dots a_5} &= (\lambda\Gamma_{a_1a_2a_3a_4 i}D_j\lambda)D^i F^j{}_{a_5} - \text{dual} \\
B^{(10)}_{a_1\dots a_5} &= (\lambda\Gamma_{a_1a_2a_3}D^i\lambda)D_i F_{a_4a_5} - \text{dual}\,.
\end{aligned} \tag{3.3}$$

A factor of α'^3 is suppressed on the right-hand side above and in the following.

A similar analysis can be performed for the most general redefinition δA_α at order α'^3. The result is,

$$\delta A_\alpha = \sum_{i=1}^{8} a_i A^{(i)}_\alpha\,, \tag{3.4}$$

where $a_1, \dots a_8 \in \mathbb{R}$. Schematically, $A^{(1)} \dots A^{(8)}$ are given by the projections of the following products of irreducible representations onto (00010):

$$\begin{aligned}
A^{(1)} &\sim F\lambda^3 \sim (01000) \otimes (00001)^3_a \\
A^{(2,3)} &\sim \lambda(D\lambda)^2 \sim (00001) \otimes (10001)^2 \\
A^{(4,5)} &\sim FDF\lambda \sim (01000) \otimes (11000) \otimes (00001) \\
A^{(6,7)} &\sim F^2(D\lambda) \sim (01000)^2_s \otimes (10001) \\
A^{(8)} &\sim DFD^2\lambda \sim (11000) \otimes (20001)\,.
\end{aligned} \tag{3.5}$$

Explicitly,

$$\begin{aligned}
A^{(1)}_\alpha &= F^{ij}(\Gamma^k\lambda)_\alpha(\lambda\Gamma_{ijk}\lambda) \\
A^{(2)}_\alpha &= (\Gamma^{ijk}\lambda)_\alpha(D_i\lambda\Gamma_j D_k\lambda) \\
A^{(3)}_\alpha &= (\Gamma^{ijk}\lambda)_\alpha(D^l\lambda\Gamma_{ijk}D_l\lambda) \\
A^{(4)}_\alpha &= (\Gamma^k\lambda)_\alpha D_k F_{ij}F^{ij} + (\Gamma^{ijk}\lambda)_\alpha D^l F_{ij}F_{lk} \\
A^{(5)}_\alpha &= -\frac{7}{4}(\Gamma^k\lambda)_\alpha D_k F_{ij}F^{ij} + \frac{1}{4}(\Gamma^{ijk}\lambda)_\alpha D^l F_{ij}F_{lk} \\
A^{(6)}_\alpha &= (\Gamma^{ijk}D^l\lambda)_\alpha F_{ij}F_{kl} \\
A^{(7)}_\alpha &= (\Gamma^i D^j\lambda)_\alpha F_i{}^l F_{jl} \\
A^{(8)}_\alpha &= D^i F^{jk}(\Gamma_k D_i D_j\lambda)_\alpha\,.
\end{aligned} \tag{3.6}$$

Finally, the most general anti-self-dual five-form spinor $\widetilde{J}_{a_1\dots a_5\alpha}$ at order α'^3 is of the form,

$$\widetilde{J}_{a_1\dots a_5\alpha} = \sum_{i=1}^{4} c_i C^{(i)}_{a_1\dots a_5\alpha}\,, \tag{3.7}$$

where $c_1, \dots c_4 \in \mathbb{R}$. Schematically, $C^{(1)} \dots C^{(4)}$ are given by the projections of the following products of irreducible representations onto (00030):

$$\begin{aligned}
C^{(1)} &\sim F\lambda^2 D\lambda \sim (01000) \otimes (00001)^2_a \otimes (10001) \\
C^{(2)} &\sim FDFD\lambda \sim (01000) \otimes (11000) \otimes (10001) \\
C^{(3)} &\sim \lambda D\lambda D^2\lambda \sim (00001) \otimes (10001) \otimes (20001) \\
C^{(4)} &\sim (D\lambda)^3 \sim (10001)^3\,.
\end{aligned} \tag{3.8}$$

Note that in the decomposition of the above products into irreducible representations, there is no (00003) component. Therefore the projection $\widetilde{S}_{a_1\dots a_5\alpha}|$ of a spinor-five-form $\widetilde{S}_{a_1\dots a_5\alpha}$ onto its gamma-traceless part is equivalent to the projection onto the (00030) (the anti-self-dual) part. We have,

$$\begin{aligned}
\widetilde{S}_{a_1\dots a_5}| = {}& \frac{1}{6}\widetilde{S}_{a_1\dots a_5} - \frac{1}{6}\Gamma_{a_1}{}^i\widetilde{S}_{a_2\dots a_5 i} - \frac{1}{12}\Gamma_{a_1a_2}{}^{ij}\widetilde{S}_{a_3a_4a_5ij} + \frac{1}{36}\Gamma_{a_1a_2a_3}{}^{ijk}\widetilde{S}_{a_4a_5ijk} + \\
& + \frac{1}{144}\Gamma_{a_1\dots a_4}{}^{ijkl}\widetilde{S}_{a_5ijkl} - \frac{1}{720}\Gamma_{a_1\dots a_5}{}^{ijklm}\widetilde{S}_{ijklm}\,,
\end{aligned} \tag{3.9}$$

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where antisymmetrization in $a_1 \ldots a_5$ is understood on the right-hand-side and we have omitted all spinor indices. We can now give an explicit form for $C^{(1)} \ldots C^{(4)}$,

$$
\begin{aligned}
C^{(1)}_\alpha &= (\Gamma^i D_{a_1}\lambda)_\alpha (\lambda\Gamma_{a_2a_3a_4}\lambda) F_{a_5 i}| \\
C^{(2)}_\alpha &= (\Gamma^i D_{a_1}\lambda)_\alpha D_i F_{a_2a_3} F_{a_4a_5}| \\
C^{(3)}_\alpha &= (\Gamma^i D_{a_1}\lambda)_\alpha (\lambda\Gamma_{a_2a_3a_4} D_{a_5} D_i\lambda)| \\
C^{(4)}_\alpha &= (\Gamma^i D_{a_1}\lambda)_\alpha (D_{a_2}\lambda\Gamma_{a_3a_4a_5} D_i\lambda)| \,.
\end{aligned}
\tag{3.10}
$$

The action of the operator D_2 can be computed in a straight-forward way using definition (2.3) and equations (2.7), (2.9). We find,

$$
\begin{aligned}
D_2 B^{(1)} &= 16 C^{(1)} \\
D_2 B^{(2)} &= -8 C^{(1)} \\
D_2 B^{(3)} &= -2 C^{(2)} \\
D_2 B^{(4)} &= 2 C^{(4)} \\
D_2 B^{(5)} &= 0 \\
D_2 B^{(6)} &= 4 C^{(2)} \\
D_2 B^{(7)} &= -2 C^{(3)} \\
D_2 B^{(8)} &= -8 C^{(2)} \\
D_2 B^{(9)} &= -16 C^{(3)} \\
D_2 B^{(10)} &= 0 \,.
\end{aligned}
\tag{3.11}
$$

We see that, with respect to the basis $(B^{(1)}, \ldots B^{(10)})$, the Kernel of the operator D_2 is the six-dimensional subspace given by,

$$
\begin{aligned}
\mathrm{Ker}(D_2) = \mathrm{Span}\{(b_1, 2b_1, 2b_6 - 4b_8, 0, b_5, b_6, -8b_9, b_8, b_9, b_{10})\}\,, \\
b_1, b_5, b_6, b_8, b_9, b_{10} \in \mathbb{R}\,.
\end{aligned}
\tag{3.12}
$$

Similarly, the action of the operator D_1 can be computed taking (2.4) into account. The result is,

$$
\begin{aligned}
D_1 A^{(1)} &= -\frac{1}{2} B^{(1)} - B^{(2)} \\
D_1 A^{(2)} &= -8 B^{(6)} - 4 B^{(8)} + \frac{2}{3} B^{(10)} \\
D_1 A^{(3)} &= -8 B^{(5)} + 8 B^{(10)} \\
D_1 A^{(4)} &= 8 B^{(3)} - 2 B^{(8)} \\
D_1 A^{(5)} &= 2 B^{(3)} + \frac{4}{3} B^{(7)} - \frac{1}{2} B^{(8)} - \frac{1}{6} B^{(9)} + \frac{1}{3} B^{(10)} \\
D_1 A^{(6)} &= -8 B^{(3)} - 4 B^{(6)} \\
D_1 A^{(7)} &= -\frac{1}{6} B^{(5)} \\
D_1 A^{(8)} &= 0 \,.
\end{aligned}
\tag{3.13}
$$

With respect to the basis $(B^{(1)}, \dots B^{(10)})$, the Image of the operator D_1 is the six-dimensional subspace given by,

$$\begin{aligned} \mathrm{Im}(D_1) = \mathrm{Span}\Big\{ \Big(& -\frac{1}{2}a_1, -a_1, 8a_4 + 2a_5 - 8a_6, 0, -8a_3 - \frac{1}{6}a_7, -8a_2 - 4a_6, \\ & \frac{4}{3}a_5, -4a_2 - 2a_4 - \frac{1}{2}a_5, -\frac{1}{6}a_5, \frac{2}{3}a_2 + 8a_3 + \frac{1}{3}a_5 \Big) \Big\}, \\ & a_1 \dots a_8 \in \mathbb{R}. \end{aligned} \tag{3.14}$$

Comparing (3.14), (3.12), it is easy to see that the Image of D_1 is in the Kernel of D_2. This of course is just the nilpotency property (1.1).

The spinorial cohomology SH can be readily computed from (3.14), (3.12),

$$SH = \frac{\mathrm{Ker}(D_2)}{\mathrm{Im}(D_1)} = 0\,. \tag{3.15}$$

The triviality of SH means that at order α'^3, there *cannot* be any supersymmetric deformation of ordinary abelian $d = 10$ SYM.

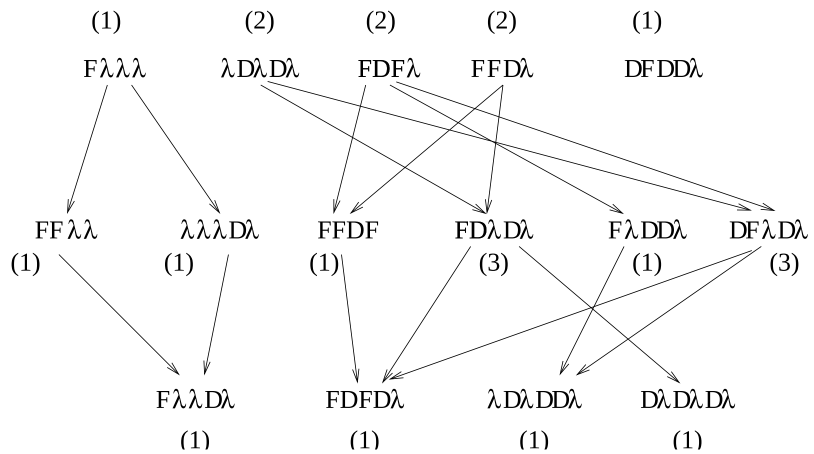

Figure 1: In the three rows we have included, schematically, the various terms in δA_α, J_5, $\widetilde{J}_5$ (equations (3.4), (3.1), (3.7), respectively). The arrows from the first to the second row indicate the action of D_1, while the arrows from the second to the third row indicate the action of D_2. The numbers in parentheses denote multiplicities.

4. Discussion

In this paper we computed the spinorial cohomology for ten-dimensional abelian SYM at order α'^3 and we found that it is trivial. We have therefore demonstrated that to this order in α' the abelian BI is the unique supersymmetric deformation of ordinary abelian SYM.

We would like to emphasize that the only assumption we make is that the theory possesses *linear* ten-dimensional supersymmetry. Consequently, any computation coming from superstrings would necessarily respect (3.15) *to all orders in the string coupling.* In particular the result of the present paper combined with the result of [25] implies that, up to order α'^3, all g_s corrections to the abelian supersymmetric BI action can be absorbed in an overall string-tension renormalization. To one-loop order in g_s, this was indeed shown to be the case in [29].

A natural generalization of this paper would be to compute the non-abelian spinorial cohomology at α'^3-order [2] for some (partial) results on the non-abelian BI at this order in α'. The bosonic part of the $\mathcal{O}(\alpha'^3)$ corrections to the effective action of type-I superstring theory at tree-level in the string coupling was presented in [33]. The action including fermions up to quadratic order was recently given in [34]. This task is very much involved technically. One can appreciate the level of complication by noting that in the non-abelian case one has to deal with products of fields valued in the adjoint of a general Lie group G. These products are contracted with G-invariant tensors of rank as high as five. The analysis of this paper would have to be repeated separately for each plethysm of each G-invariant tensor and there are generically twenty-six non-vanishing plethysms in each fifth-rank tensor!

Another more feasible, perhaps, generalization is the computation of the abelian spinorial cohomology at order α'^4. It would be interesting to see whether the vanishing of SH persists to this order. Note that $SH = 0$ would imply that there is no new independent superinvariant arising at order α'^4; it would *not* imply $F_{\alpha\beta} = 0$ (Equivalently: $J_5 = 0$). In [20, 25], the most general $F_{\alpha\beta}$ allowed by linear supersymmetry was determined to order α'^2. As was explained in these references, the constraint $D_2 J_5 = 0$ is satisfied to order α'^2, but not to order α'^4. To ensure that the constraint is satisfied to this order, one would need to compensate by adding an α'^4 correction to $F_{\alpha\beta}$. The $\partial F \to 0$ limit of this correction should coincide (presumably after field redefinitions) with the recent result of [35].

The techniques of this paper can also be applied to eleven-dimensional supergravity [36]. It would be interesting to obtain the spinorial cohomology for the first few orders in the Planck-length (long-wavelength) expansion and thereby determine the possible deformations of the theory.

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